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Quiz 1: Supplementary Question

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A screenshot of a social media post

Description automatically generatedx\_high = 6, F(x\_high) = ~ -0.8, Negative

From this, we would need an x\_low for which F(x\_low) would be positive. Also, we need to take into account that we want our algorithm to converge upon the root denoted by alpha.

Any value of x\_low that is to the right of the root alpha would be a good choice…if the y-values to the right of the root alpha were partially negative.

This brings us to a dilemma. From close inspection, we can tell that it is almost impossible to pick an “approximate” value for our possible candidate for x\_low because without knowing exactly where x\_low lies on the x-axis, there is a very high possibility, daresay an unavoidable possibility, that our configuration would never converge onto the root alpha.

We would always have to replace x\_low with a value on the x-axis that is going to get further and further away from the root alpha while our x\_high stays the same at x\_high = 6 due to the abundance of positive y-values to the right and to the left of the root alpha.

Therefore, we would need an exact value of the root alpha. However, I can give you my best approximation.

Let’s say that the root alpha lies at exactly the value x\_alpha. Then, it is clear that to avoid converging away from alpha, we need x\_mid to equal x\_alpha exactly.

x\_mid = x\_alpha = (x\_high + x\_low)/2

2 \* x\_alpha = x\_high + x\_low

2(x\_alpha) – x\_high = x\_low

Pligging in x\_high = 6…

2(x\_alpha) – 6 = x\_low

Now, we know from the documentation of the question that f(x) = 0 but does not cross 0 at our desired root where 2 < alpha < 3.

Now we are getting somewhere. We can plug in our values that could possibly be the value for x\_alpha and get an interval for where our x\_low should lie.

Plugging in x\_alpha = 2:

2(2) – 6 = x\_low

4 – 6 = x\_low

x\_low = -2

Plugging in x\_alpha = 3:

2(3) – 6 = x\_low

6 – 6 = x\_low

x\_low = 0

From this, we can see that our exact value for x\_low for our algorithm to converge unto the root alpha, with x\_high at 6, resides in the interval -2 < x\_low < 0. This is the closest we can get to the approximation of x\_low without knowing the function f(x) or knowing where the root alpha truly lies on the graph.

***\*\*\*Equation here\*\*\****

2(2(x\_alpha) - 6) – 6 = 4(x\_alpha) – 18 = x\_low

2(2(2(x\_alpha – 6) – 6) – 6 = 8(x\_alpha) – 42 = x\_low

2(8(x\_alpha) – 42) – 6 = 16(x\_alpha) – 90 = x\_low

As we can see, there is a pattern.

The multiplier preceding the value for x\_alpha increases by a factor of 2 every iteration.

The difference at the end increases by some factor of 6.

0, 6, 18, 42, 90

0 = 6 \* 0

6 = 6\* 1 + 0

18 = 6 \* 2 + 6 \* 1

42 = 6 \* 4 + 6 \* 2 + 6 \* 1

As we can see, the difference increases by a factor of 2, added to the previous differences.

We can denote this in a sum notation

where n is the nth iteration of x\_mid away from x\_alpha

We can simplify the equation to

2^n(x\_alpha) - where 0 <= n < ∞